Jim Garven

From:	noreply+feedproxy@google.com on behalf of Options, Futures, Derivatives <jgarven@gmail.com>
Sent:	Sunday, March 06, 2011 1:21 AM
To:	jimgarven@gmail.com
Subject:	Options, Futures, and Other Derivatives Course Blog

Options, Futures, and Other Derivatives Course Blog

Some observations concerning CRR versus other approaches to binomial option pricing

Posted: 05 Mar 2011 01:53 PM PST

During the past few lectures, we have seen how options can be priced using delta hedging, replicating portfolio, and risk neutral valuation approaches. All of these perspectives are worthwhile since they enable us to think carefully as well as deeply concerning the economics of option pricing. For example, the delta hedging approach illustrates that an appropriately hedged portfolio consisting of either long-short call-share positions or long-long put-share positions is riskless and consequently must produce a riskless rate of return; taking the share price as given, any other outcome suggests that options are mispriced. Similarly, the replicating portfolio approach reminds us that a call option represents a “synthetic” margined stock investment, whereas a put option represents a “synthetic” short sale of the share combined with lending money; thus, if there are any differences between the options and their replicating portfolios, this also indicates that the options are mispriced. Finally, the notion of **arbitrage-free pricing** itself implies that a risk neutral valuation relationship exists between the option and its underlying, which in turn enables us to calculate risk neutral probabilities.

One feature that is common to the delta hedging, replicating portfolio, and risk neutral valuation approaches is that one finds today’s option price by starting at the very end of the binomial tree and working step-by-step, node-by-node back to the very beginning of the tree. The unique feature of the Cox-Ross-Rubinstein (CRR) model is that it provides us with a very convenient analytic short-cut; specifically, it gets rid of the need for backward induction. CRR enables us to focus our attention on that subset of nodes at the very end of the tree where the option payoffs are positive, and then we employ risk neutral valuation in order to obtain the current option price. The “trick” here is to determine which nodes feature positive option payoffs. Specifically, we know that the minimum number of “up” moves required during the course of n timesteps is given by the parameter a, where a is the smallest (non-negative) integer that is greater than $\ln(X/Sd^n)/\ln(u/d)$. If $a = 0$, this means that all of the call option payoffs at the end of the tree are positive, and that all of the payoffs for an otherwise identical put option are equal to zero. If $a = n$, then the only node at which a call pays off is when there has been n consecutive up moves. In theory, a can exceed n; in this case, the put pays off at all end-of-tree nodes, whereas the call is always out of the money.

The CRR call option pricing equation is:

$$c = S \cdot \left[\sum_{j=a}^{n} \left(\frac{n!}{j!(n-j)!} \right) \cdot q^{j} \cdot (1-q)^{n-j} \cdot \left(u^{j} \cdot d^{n-j} \cdot e^{-rn\delta t} \right) \right]$$

$$- Xe^{-rn\delta t} \left[\sum_{j=a}^{n} \left(\frac{n!}{j!(n-j)!} \right) \cdot q^{j} \cdot (1-q)^{n-j} \right],$$

and the CRR put option pricing equation is:

$$p = Xe^{-rn\delta t} \left[\sum_{j=0}^{a-1} \left(\frac{n!}{j!(n-j)!} \right) \cdot q^{j} \cdot (1-q)^{n-j} \right]$$

$$-S \cdot \left[\sum_{j=0}^{a-1} \left(\frac{n!}{j!(n-j)!} \right) \cdot q^{j} \cdot (1-q)^{n-j} \cdot \left(u^{j} \cdot d^{n-j} \cdot e^{-rn\delta t} \right) \right]$$

During last Thursday's lecture, I numerically illustrated special cases of the CRR equations for puts and calls using the following parameter values: S = 100, X = 100, u = 1.05, d = .95, dt = 1/12, q = .5418, and r = .05. Here's the tree of stock prices implied by these parameters:

				121.55
			115.76	
		110.25		109.97
	105.		104.74	
100		99.75		99.50
	95.		94.76	
		90.25		90.02
			85.74	
				81.45

The special cases involved calculating call and put prices for n = 1, 2, 3, and 4. This is all shown on pp. 39-47 of my **Binomial Trees** lecture note. Here's a summary of what you'll find there:

1. n = 1. Then ln(X/Sd^n)/ln(u/d) = 0.51, which implies that a = 1, since 1 is the smallest integer which is greater than 0.51. Furthermore, a = n = 1 implies that the only node at which a call option is in the money is at node u (since the call option will only be in the money after one up move and when there is only one timestep), and the only node at which a put option is in the money is at node d (since the put option will only be in the money after one down move and when there is only one timestep), Thus, the CRR call and put pricing equations are c = exp(-rdt)[q(uS-X)] = 0.9958(0.5418)(5) =$2.70 and p = exp(-rdt)[(1-q)(X-dS)] = 0.9958(0.4582)(5) =$2.28.

2. n = 2. Then ln(X/Sd^n)/ln(u/d) = 1.03, which implies that a = 2, since 2 is the smallest integer which is

greater than 1.03. Furthermore, $a = n = 2$ implies that the only node at which a call option is in the money is at node uu (since the call option will only be in the money after two up moves and when there are only two timesteps). On the other hand, the put option will be in the money at nodes dd and ud). Thus, the CRR call and put pricing equations are $c = exp(-r2dt)[q^2(u^2(S)-X)] = 0.9958^2(0.5418^2)(10.25) =$ \$2.98 and $p = exp(-r2dt)[(1-q)^2(X-d^2(S)) + 2q(1-q)(X-udS)] = .9958^2[(0.4582^2)(9.75) + 2(.5418)(.4582)(.25)$ =\$2.15.

3. $n = 3$. Then $\ln(X/Sd^n)/\ln(u/d) = 1.54$, which implies that $a = 2$, since 2 is the smallest integer which is greater than 1.54. Furthermore, $n = 3 > a = 2$ implies that the call option will be in the money at all nodes involving 2 and 3 up moves; i.e., at nodes uud and uuu. On the other hand, the put option will be in the money at nodes involving no up moves (i.e., node ddd) and one up move (i.e., node udd). Thus, the CRR call and put pricing equations are $c = exp(-r3dt)[3q^2(1-q)(u^2(dS)-X) + q^3(u^3(S)-X)] = 0.9958^3[3(0.5418^2)(.4582)(4.74)+ (0.54183^3)(15.76)=$ \$4.36 and $p = exp(-r3dt)[3q^2(1-q)(u^2(dS)-X) + q^3(u^3(S)-X)] = 0.9958^3[(0.4582^3)(14.26)+ 3(0.45822^2)(.5418)(5.24)=$ \$3.12.

4. $n = 4$. Then $\ln(X/Sd^n)/\ln(u/d) = 2.05$, which implies that $a = 3$, since 2 is the smallest integer which is greater than 2.05. Furthermore, $n = 4 > a = 3$ implies that the call option will be in the money at all nodes involving 3 and 4 up moves; i.e., at nodes $uuud$ and $uuuu$. On the other hand, the put option will be in the money at nodes involving no up moves (i.e., node $dddd$), one up move (i.e., node $uddd$) and two up moves (i.e., node $uudd$). Thus, the CRR call and put pricing equations are $c = exp(-r4dt)[4q^3(1-q)(u^3(dS)-X) + q^4(u^4(S)-X)] = 0.9958^4[4(0.5418^3)(.4582)(9.97)+ (0.5418^4)(21.55) =$ \$4.68 and $p = p = exp(-r4dt)[(1-q)^4(X-d^4(S)) + 4q(1-q)^3(X-ud^3(S)) + 6q^2(1-q)^2(X-u^2(d^2)(S))] = 0.9958^4[(0.4582^4)(18.55)+4(0.4582^3)(0.5418)(9.98)+6(0.4582^2)(0.5418^2)(0.50)=$ \$3.03.

The final (and very important point) that I made on Thursday was that the Black-Scholes-Merton (BSM) call and put pricing equations represent “limiting” cases of CRR, where we allow the number of timesteps to become arbitrarily large while the length of each timestep become arbitrarily small. Conceptually, pricing options with BSM involves the same logic as CRR – however, in the case of BSM, we focus our attention on the continuum of stock prices that can exist on the option expiration date and simply calculate the present value of the risk neutral expected value of these payoffs. Shortly after you get back from spring break, we’ll tackle the BSM pricing model. Since BSM pricing involves delta hedging and replicating portfolios over infinitesimally small units of time, BSM is often referred to as an example of “continuous time” finance.

Email this

Prof. Myron Scholes, 1997 Nobel Prize Winner in Economic Sciences, Delivers “Last Lecture”

Posted: 05 Mar 2011 12:42 PM PST

From **YouTube**: “Myron Scholes, the Frank E. Buck Professor of Finance, Emeritus, delivers a speech as part of the business school’s Last Lecture Series. Each year, MBA students invite members of the faculty to deliver a Last Lecture, touching on a topic that is important to the speaker, that sums up an area of research or academic work, or that may be a parting word of advice or inspiration to students."

This video is well worth watching. Professor Scholes lectures on topics such as liquidity, risk transfer, chaos, the role of hedge funds, and how these various topics are interrelated. Considering that this video was recorded in 2004, his lecture seems remarkably prescient in terms of providing important and relevant perspectives on the Financial Crisis of 2007–????.

Email this

This posting includes an audio/video/photo media file: **Download Now**

You are subscribed to email updates from Options, Futures, Derivatives
To stop receiving these emails, you may unsubscribe now.

Email delivery powered by Google

Google Inc., 20 West Kinzie, Chicago IL USA 60610